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Models for Predicting Volatilization of Soil-Incorporated Pesticides[1]

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ABSTRACT

In the absence of appreciable mass transfer due to water movement, diffusion processes in the soil account for the movement of pesticides to the soil surface to replace that lost by volatilization. Published solutions for heat flow equations have been applied to the volatilization of lindane and dieldrin from Gila silt loam for a number of different initial and boundary conditions. Predicted fluxes agreed well with experimental values. Five models have been proposed to describe various environmental conditions found in the field. Models I, II, and III assume pesticide concentration at the soil surface is maintained at zero concentration by air movement. Model IV assumes surface pesticide concentrations greater than zero with air turbulence sufficient to maintain zero pesticide concentration gradient in the air above the soil. Model V assumes a nonmoving air layer of various depths above the soil surface so that the pesticide concentration gradient in the air controls the rate of volatilization.

Additional Index Words: dieldrin, lindane, organochlorine insecticides, diffusion, transport processes.

SOME ORGANIC compounds, especially pesticides, applied to soils are frequently transported to nontarget areas. In order to predict losses from soil, it is desirable to know something of the mechanisms involved in the transport of pesticides through the soil. This paper will concentrate on loss from soil by volatilization. Several investigators have demonstrated that the volatility rate for soil-incorporated pesticides is initially high and decreases with time at a rate dependent on the pesticide and on the experimental conditions (Farmer et al., 1972; Igue et al., 1972; Spencer and Cliath, 1973).

As a pesticide volatilizes at the soil surface, it may be replaced by pesticides within the soil profile. Pesticides move to the soil surface by mass transport in water flow to the surface, or by diffusion of the pesticide. Farmer et al. (1973) has shown the importance of diffusion processes in controlling dieldrin volatility.

The objective of this study is to develop mathematical equations for predicting volatilization of soil-incorporated pesticides as a diffusion-controlled process and to compare calculated with measured results.

THEORETICAL CONSIDERATIONS

The basic assumption in the mathematical treatment of the movement of pesticides in soils under a concentration gradient is the applicability of the diffusion laws. The changes in pesticide concentration within the soil as well as the loss of pesticides at the soil surface by volatilization can then be predicted by solving the diffusion equation for different boundary conditions. Recognizing the analogy between the heat transfer equation (Fourier's law) and the transfer of matter under a concentration gradient (Fick's law), solutions of the heat transfer equation given by the mathematical theory of conduction of heat may be used. The mathematical model for predicting volatilization of pesticides is then given as a set of boundary conditions sufficient to solve the diffusion equation.

Consider a system where a pesticide is uniformly mixed with a layer of soil and volatilizing at the soil surface. If diffusion is the only mechanism supplying pesticide to the surface, and if we assume isotropic conditions in the soil as well as constancy of the diffusion coefficient D, then the general diffusion equation is

$$\frac{\partial^2 c}{\partial x^2} - \frac{1}{D}\frac{\partial c}{\partial t} = 0 \qquad [1]$$

where c is the pesticide concentration in the soil (g/cm^{-3} total volume), x is the distance measured normal to the soil surface (cm), D is the diffusion coefficient ($cm^2\ sec^{-1}$), and t is the time (sec).

Model I

As a first approximation we may assume that pesticide volatilizes and is removed rapidly maintaining a zero concentration at the soil surface. The depth of the soil layer treated uniformly with pesticide is L and the initial concentration C_o (g/cm^3). The initial and boundary conditions for which Eq. [1] is to be solved are then

$$c = C_o \text{ at } t = 0,\ 0 \le x \le L$$
$$c = 0 \text{ at } x = 0 \text{ and } t > 0$$
$$\partial c/\partial x = 0 \text{ at } x = L.$$

Carslaw and Jaeger [1959, p. 97, Eq. (8)] give a solution for these boundary conditions that can be used by accepting the following analogies:

v (temperature) = c (concentration)
V_o (initial temperature) = C_o (initial concentration)
κ (thermal diffusivity) or K (thermal conductivity) = D (apparent diffusion coefficient).

The concentration units for c and C_o must be expressed on a total volume basis in order for D to be analogous to K and κ.

The solution for Eq. [1] is then

$$c = \frac{4C_o}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}\{\exp[-D(2n+1)^2\pi^2 t/4L^2]\}\cos\frac{(2n+1)\pi(L-x)}{2L}. \qquad [2]$$

(Carslaw and Jaeger used the boundary condition $x = L$ and we have used $x = 0$ at the soil surface. Identical results are achieved by our using $(L - x)$ in the equations where Carslaw and Jaeger used x). The pesticide flux, f ($g/cm^2/sec$), through the surface is given as the concentration gradient at $x = 0$ times the diffusion coefficient D

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$$f = D[\partial c/\partial x]_{x=0}$$

$$= \frac{DC_o}{(\pi Dt)^{1/2}} [1 + 2 \sum_{n=1}^{\infty} (-1)^n \exp(-n^2L^2/Dt)]. \quad [3]$$

Model II

The summation term in Eq. [3] decreases with increasing L and decreasing D and t. If this term is small enough to be negligible, Eq. [3] reduces to

$$f = DC_o/(\pi Dt)^{1/2}. \quad [4]$$

This equation is identical with the solution given by Carslaw and Jaeger [1959, p. 59, Eq. (3)] for heat flow in an infinite solid. The concentration for the semi-infinite case is given by

$$c = C_o \operatorname{erf} [x/2(Dt)^{1/2}]. \quad [5]$$

Equations [4] and [5] are applicable also on a finite system (in the region $0 < x < L$) as long as the concentration at the lower boundary of the soil layer, $x = L$, is not decreased by pesticide moving in the upward or downward direction. To estimate the maximum time at a given set of parameters for which Model II is adequate, let us assume the critical value to be a drop of 1% in the initial concentration, C_o, at the lower boundary of the soil layer. With this assumption the boundary conditions used in deriving Eq. [5] are violated if

$$\operatorname{erf} [L/2(Dt)^{1/2}] \leq .99$$

or

$$t > L^2/14.4\ D.$$

Model III

Model I assumed that no diffusion occurs across the lower boundary $x = L$. This boundary condition is appropriate for laboratory studies which will be reported later, but not realistic for field conditions where diffusion can occur downward across the boundary $x = L$. The boundary conditions for the latter case are

$$c = C_o \text{ at } t = 0,\ 0 \leq x \leq L$$

$$c = 0 \text{ at } t = 0,\ x > L$$

$$c = 0 \text{ at } t > 0,\ x = 0$$

Solution of Eq. [1] with these initial and boundary conditions is given by Carslaw and Jaeger (1959, p. 62, Eq. [14]) as

$$c = (C_o/2) \{2 \operatorname{erf} [x/2(Dt)^{1/2}] - \operatorname{erf} [(x-L)/2(Dt)^{1/2}] - \operatorname{erf} [(x+L)/2(Dt)^{1/2}]\}. \quad [6]$$

The flux is obtained by differentiating Eq. [6] with respect to x, determining $\partial c/\partial x$ at $x = 0$, and multiplying by D. The result is

$$f = [DC_o/(\pi Dt)^{1/2}] [1 - \exp(-L^2/4Dt)]. \quad [7]$$

Note that Eq. [7] reduces to Eq. [4] for large values of $L^2/4Dt$. Less than 1% error will result from using Eq. [4] if

$$\exp(-L^2/4Dt) < .01$$

or

$$t < L^2/18.4D.$$

Model IV

The solutions for the diffusion equation become more difficult if the concentration at the soil surface is variable instead of being maintained at zero. The rate of removal of pesticide in the air layer above the soil may then become a limiting factor in the rate of volatilization.

A model which accounts for a possible incomplete depletion at the soil surface may be described as follows. A soil layer ($0 \leq x < L$) has a uniform initial distribution of pesticide C_o with no flux allowed through the lower boundary at $x = L$. The soil surface, $x = 0$, is in contact with a volume, V, of well stirred air (uniform concentration within the air) from which a volume per unit time and unit area, v, is withdrawn and replaced by the same volume of air at zero concentration. The airflow thus removes an amount of pesticide which is equal to the flow velocity, v, times the concentration within the air, c_a. The initial and boundary conditions are

$$c = C_o \text{ at } t = 0,\ 0 \leq x \leq L$$

$$\partial c/\partial x = 0 \text{ at } x = L$$

$$f = v\, c_a \text{ at } t > 0,\ x = 0.$$

This model is analogous to that of Carslaw and Jaeger (1959, p. 129, Eq. [11]) in which a slab of uniform initial temperature is in contact with a mass per unit area of well-stirred fluid which loses heat by radiation at a rate H times its temperature. The initial temperature of the fluid is zero C. Their solution is

$$c = 2C_o \sum_{n=1}^{\infty} \frac{\exp(-D\alpha_n^2 t)(h - k\alpha_n^2) \cos [\alpha_n (L-x)]}{[L(h-k\alpha_n^2)^2 + \alpha_n^2 (L+k) + h] \cos \alpha_n L} \quad [8]$$

where α_n are the roots of

$$\alpha \tan (\alpha L) = h - k\alpha^2. \quad [9]$$

Carslaw and Jaeger define $h = H/K$ where K is the thermal conductivity. For our case K is replaced by the diffusion coefficient, D, and H by the air flow velocity, v.

The concentration in the air, c_a, must be expressed in terms of the concentration at the soil surface, c_s. This is done by the use of the adsorption isotherm. Adsorption isotherms for lindane and dieldrin given in literature (Spencer and Cliath, 1970; Spencer, Cliath and Farmer, 1969) suggests that we may, for a small concentration range of c_s consider the isotherm as

$$c_a = R\, c_s. \quad [10]$$

Thus, h in Eq. [8] appears to be

$$h = Rv/D. \quad [11]$$

(Note that both concentrations c_a and c_s in Eq. [10] must be expressed on a total volume basis.)

The term k, appearing in Eq. [8] is defined by Carslaw and Jaeger as the ratio of the heat content of the fluid to the heat content of the slab. The ratio in the heat flow model is equal to R in Eq. [11] so that

$$k = R. \quad [12]$$

It can be shown that for all cases treated here

$$k << h \text{ or } k\,\alpha_n^2 << h.$$

Equations [8] and [9] can therefore be simplified to

$$c = 2C_o \sum_{n=1}^{\infty} \frac{\exp(-D\alpha_n^2 t)\, h \cos[\alpha_n (L-x)]}{(Lh^2 + \alpha_n^2 L + h)\cos \alpha_n L} \quad [13]$$

and

$$\alpha \tan(\alpha L) = h. \quad [14]$$

The pesticide flux through the soil surface at $x = 0$ is then given by

$$f = 2DC_o \sum_{n=1}^{\infty} \frac{\exp(-D\alpha_n^2 t)\, h^2}{Lh^2 + \alpha_n^2 L + h}. \quad [15]$$

Carslaw and Jaeger (1959, p. 491) give a table of the first six roots of the equation

$$\alpha \tan \alpha = M \quad [16]$$

which can be used instead of Eq. [14] when $L = 1$ or by scaling all variables containing units of length in terms of L.

Model V

Model IV assumes the air above the soil to be sufficiently stirred so that its concentration may be taken to be constant throughout. This may be due to turbulence as well as to a large diffusion coefficient within the air. However, Model IV does not take into account any diffusive transfer of pesticide. When air velocity becomes zero, the predicted flux through the soil surface becomes zero as well. This model thus can only be appropriate if the convective flow due to air movement is considerably greater than the flow due to a diffusion gradient.

If there is a non-moving air layer in contact with the soil surface, the following model may be adequate. The diffusion coefficient of a pesticide in air is D' and the thickness of the air layer is d. As a first approximation we may neglect the capacity of the air layer. For example, Spencer and Cliath (1970) report that at 30C a soil with 10^4 ng/cm^3 lindane is in equilibrium with a lindane vapor density in air of 0.22 ng/cm^3. Then, if the concentration in soil at the soil surface is c and the concentration in air at the soil surface is Rc, we have the flux through the air layer given by

$$f = D'\,Rc/d$$

when the concentration at the upper edge of the air layer is zero. The initial and boundary conditions are

$$c = C_o \text{ at } t = 0,\ 0 \le x \le L$$

$$\partial c/\partial x = 0 \text{ at } x = L$$

$$f = (D'\,Rc_s)/d \text{ at } t > 0,\ x = 0$$

$$c = c_s \text{ at } t > 0,\ x = 0.$$

These conditions are equivalent to the "radiation" boundary conditions described by Carslaw and Jaeger (1959). We may then adopt the solution given by these authors [p. 316, Eq. (24)] after substituting h in their equations by

$$h = D'\,R/Dd. \quad [17]$$

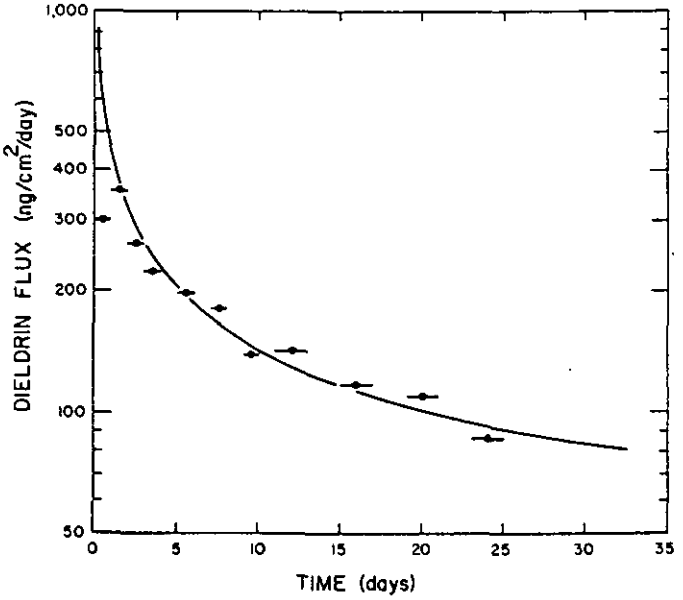

Fig. 1—Comparison between calculated values (*solid curve*) from Model II and experimental values (*horizontal lines*) for dieldrin flux. The experimental values were taken from Spencer and Cliath (1973). The length of the horizontal lines indicate the time over which the experimental values were taken.

The concentration of pesticide in soil is

$$c = \frac{2D'RC_o}{Dd}$$

$$\sum_{n=1}^{\infty} \frac{\exp(-D\alpha_n^2 t)\cos[\alpha_n (L-x)]}{[L(D'R/Dd)^2 + L\alpha_n^2 + D'R/Dd]\cos \alpha_n L} \quad [18]$$

with

$$\alpha \tan(\alpha L) = D'R/Dd. \quad [19]$$

The flux through the soil surface at $x = 0$ is

$$f = 2DC_o \sum_{n=1}^{\infty} \frac{\exp(-D\alpha_n^2 t)(D'R/Dd)^2}{L(D'R/Dd)^2 + L\alpha_n^2 + D'R/Dd}. \quad [20]$$

Note that Eq. [18], [19], and [20] are identical with Eq. [13], [14] and [15], but instead of

$$h = Rv/D$$

we have defined h by Eq. [17].

RESULTS

A comparison will now be made between calculated results from our equations and results of published volatilization experiments. All these experiments, using lindane and dieldrin, were designed to measure the volatilization of pesticides from the surface of a soil column by leading an air stream over the surface. To prevent net loss of soil water and thus prevent transport of pesticide by mass flow to the soil surface, the air was brought to 100% relative

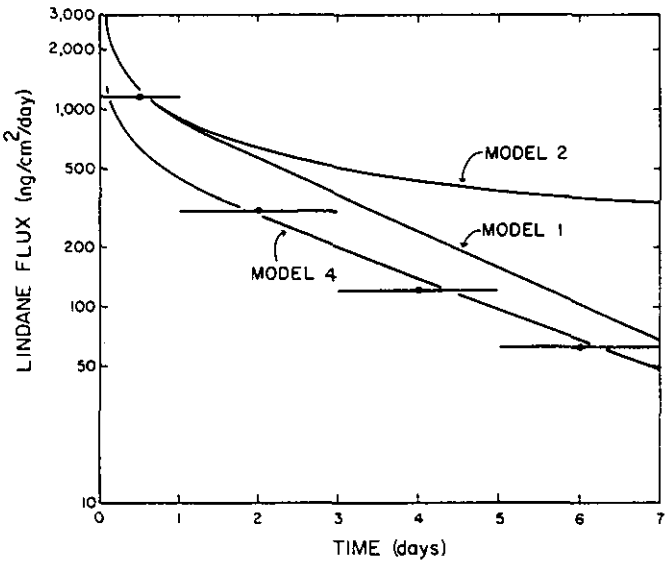

Fig. 2—Comparison between calculated and experimental values for lindane flux from treated Gila silt loam. Experimental data are from Farmer et al. (1972).

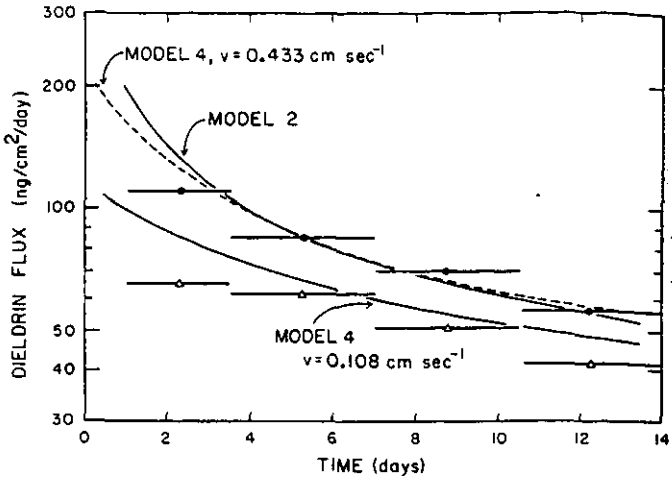

Fig. 3—Comparison of calculated and experimental values for dieldrin volatilization flux from uniformally treated Gila silt loam for air moving over the soil surface at two velocities. Experimental data from Farmer et al. (1972).

humidity. The amount of pesticide carried by the air stream was measured for distinct periods of time.

The rate of volatilization of dieldrin measured by Spencer and Cliath (1973) is presented in Fig. 1. The soil was Gila silt loam at a bulk density of 1.4 g/cm^3. The initial concentration, C_o, in the soil was 1.4×10^4 ng/cm^3 (10 ppm). Soil water content was maintained at 23% (w/w). The temperature was kept at 30C. Igue (1969) gives an apparent diffusion coefficient of $D = 2.3$ mm^2/week for dieldrin under these conditions. Air flow velocity was maintained at 2.15 cm sec^{-1}. The depth of the column was 11 cm. According to the evaluations given for Model II, we may assume the column to be infinite in length for at least 2×10^3 days after the start of the experiment. The solid curve in Fig. 1 was calculated according to Eq. [4]. The good agreement between measured and calculated values indicates that zero concentration at the soil surface is, under these conditions, a reasonable assumption. For very short times after the start of volatilization, the boundary conditions are violated in that the pesticide concentration at the soil surface is greater than zero. This may lead to some over estimation of the initial volatility rates. The low initial measured flux as compared to the calculated flux may be due to the experimental technique (e.g., losses by adsorption on walls).

Figure 2 represents data taken from a volatilization experiment carried out with lindane by Farmer et al. (1972). Parameters for this experiment were: Gila silt loam, soil water content 10% (w/w), bulk density 0.75 g/cm^3, temperature 30C, initial concentration 7.5×10^2 ng/cm^3 (10 ppm), and depth of the column 0.5 cm. With the diffusion coefficient for lindane under these conditions equal to $D = 30$ mm^2/week (taken from Ehlers et al., 1969), we calculate that after 22 hours from the start of the experiment the concentration at the bottom of the column decreases by more than 1%. This accounts for the considerable difference between the two solid curves in Fig. 2, calculated from Model I and Model II.

Figure 2 also shows a curve calculated from Model IV. The air velocity across the surface of the column was $v = 0.433$ cm sec^{-1}. R, as calculated from the adsorption isotherms given by Spencer and Cliath (1970), was $R = 3 \times 10^{-5}$. Therefore $h = R\,v/D$ becomes equal to 26.1 cm^{-1} compared to $k = R = 3 \times 10^{-5}$, the Eq. [13], [14], and [15] being applicable. The agreement between values calculated from Model IV and the measured values is quite good at the longer times and indicates that the flow velocity was not sufficient to create a zero concentration at the surface.

Volatilization experiments were conducted with dieldrin by Farmer et al. (1972) in which different air velocities were used. The soil was a Gila silt loam, soil water content 10% (w/w), bulk density 0.75 g/cm^3, initial dieldrin concentration was 7.5×10^3 ng/cm^3 (10 ppm), depth of the column was 0.5 cm, and temperature was kept at 20C. The diffusion coefficient of 1.5 mm^2/week was derived from Igue (1969), assuming a reduction of 50% compared to the value given by this author to account for the lower temperature (20C instead of 30C). A similar reduction with decreasing temperature was found by Ehlers et al. (1969) for lindane. R was taken from Spencer, Cliath, and Farmer (1969) to be 2.4×10^{-6}. With air velocities of 0.443 and 0.108 cm sec^{-1}, h became 39.46 and 9.86 cm^{-1}, respectively. Again, Eq. [13], [14], and [15] apply to this situation to allow calculation of pesticide flux.

Figure 3 shows the measured rates of dieldrin volatilization at two different air velocities together with the curves calculated from Model IV as well as a curve calculated for Model II. Under the conditions given in the experiment, it can be seen at the higher air flow velocity the boundary condition "zero concentration at the soil surface" is appropriate. This is not true for the lower air flow velocity where Model IV better accounts for the lower volatilization rates.

Note that the calculated curves for Model II and Model IV at the higher air velocity are very similar. Model IV predicts a lower flux during the initial time period than Model II. This is reasonable because initially there is pesti-

cide at the soil surface and a higher air velocity would be necessary to maintain a concentration equal to zero at the surface which is required by Model II. As time progresses and volatilization has occurred, the surface is depleted of pesticide and the surface concentration can be kept at zero with a lower air velocity. The low fluxes measured, as compared to the calculated, in the beginning of the experiment are probably due in part to losses by adsorption on walls.

DISCUSSION

From the equations presented for Model I and II, it can be seen that for a given initial concentration and depth of the soil layer, the flux at any time depends only on the apparent diffusion coefficient. Since the apparent diffusion coefficient depends on various parameters including bulk density, water content, concentration, temperature, and adsorption (Ehlers et al., 1969; Farmer and Jensen, 1970), all equations presented can only be used if during the time of observation, D remains constant. Ehlers et al., (1969) showed for lindane that this assumption holds true up to concentrations of about 20 ppm. This agreement between measured and calculated pesticide fluxes, if an appropriate model is chosen, indicates that constancy for the conditions met in the experiments may also be assumed for the dieldrin diffusion coefficient.

Models I, II, and IV prove to be valid for describing volatilization losses from experiments carried out in the laboratory with appropriate boundary conditions. Prediction of volatilization rates of pesticides under field conditions with these or similar models becomes more difficult, mainly because the boundary conditions are not as well defined as in a laboratory experiment. Yet, the models are valuable at least in making reasonable estimates of pesticide losses by diffusion.

Let us take, for example, a pesticide incorporated in a soil surface layer. If the soil surface is in contact with a moving layer of air, and if diffusion to the surface is the only transport mechanism for pesticide in the soil, then Model IV should apply. The wind velocity can be measured. One difficulty is that the model assumes that the air increment passing over each unit area of surface is initially devoid of pesticide. When a large land area is treated with pesticide, the wind will become enriched with pesticide as it passes over the field.

The results presented in Fig. 1 and 3 indicate that only very low wind velocities (less than about 0.016 km/hr^{-1} or 0.04 mi/hr^{-1}) are required to keep the pesticide concentration at zero at the soil surface. Thus it would appear Models I, II, or III could be used with considerable confidence if there is much air movement.

The situation is different if we have a non-moving air layer above the surface, for example where the air movement is restricted by a standing crop. Under these conditions Model V applies.

Figure 4 gives some lindane flux curves calculated for different values of h. The diffusion coefficient D' for lindane in air is not known, but may be derived [Ehlers et al. (1969), p. 502, Eq. (9)] as being 10^3 cm^2/day. The initial concentration was assumed to be 10^4 ng/cm^3, the diffusion

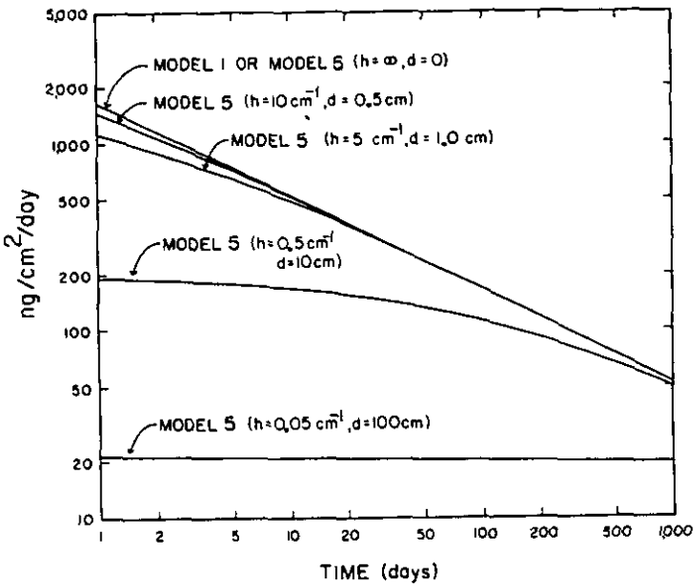

Fig. 4—Calculated values for the influence of depth of non-moving air layer on lindane volatility flux from uniformally treated Gila silt loam.

coefficient for lindane in the soil 20 mm^2/week, R as used above was 3×10^{-5}. The depth of the soil layer treated with pesticide was assumed to be 10 cm. Equation [17] then gives the values of h which correspond with different values of d, the thickness of the air layer.

From Figure 4 it can be seen that a non-moving air layer may restrict the rate of volatilization considerably. Finally it may be mentioned that since we have under most conditions water movement in the soil profile, the models presented above predict volatilization rates from a soil surface better for moderately to strongly adsorbed pesticides of low mobility with moving water.

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Time-Dependent Linearized Infiltration: II. Line Sources[1]

D. O. Lomen and A. W. Warrick[2]

ABSTRACT

Water flow from line sources is analyzed using a linearized form of the moisture flow equation. Both single and parallel line sources are considered. Results are particularly relevant for high-frequency irrigation, such as by trickle sources, for which the soil moisture at any particular point varies over a relatively small range. Numerical calculations include lines of constant matric flux potential (or equal moisture content) as a function of time and the time-dependent response to a cyclic input. Although the results are developed for surface sources, the analysis may easily be extended to buried sources.

Additional Index Words: soil water, trickle irrigation, transient flow, unsaturated flow.

In an earlier paper (Warrick, 1974) time-dependent point sources were considered using the matric flux potential ϕ of Gardner (1958), Philip (1968, 1969, 1971), Wooding (1968) and Raats (1970, 1971, 1972):

$$\phi = \int_{-\infty}^{h} K(h)dh = K/\alpha \qquad [1]$$

with

$$K = K_o \exp(\alpha h)$$

$$\alpha/k = d\theta/d\phi, \qquad k = dK/d\theta \qquad [2]$$

In Eq. [1] and [2], K is the hydraulic conductivity (units L/T), h the pressure head (L), θ the volumetric water content (dimensionless), K_o the saturated hydraulic conductivity (L/T), and α an empirical constant ($1/L$). The units of ϕ are L^2/T. With k (units L/T) of Eq. [2] a constant, the linearized form of the moisture flow equation is

$$\partial\phi/\partial t = (k/\alpha)\nabla^2\phi - k(\partial\phi/\partial z) \qquad [3]$$

with the z-axis chosen positively downwards.

In this paper, we consider the solution of Eq. [3] where a line source of strength q (units L^2/T) lies on the soil surface along the y axis. Such a geometry results, for example, when trickle emitters are closely spaced in a line or for irrigation from a porous pipe. The companion paper to this one (Warrick, 1974) is applicable when emitters are too far apart to consider as a continuous line source.

The initial condition on ϕ is taken to be zero ($h = -\infty$) and the boundary condition chosen such that no vertical flow is permitted except along the y-axis (the location of the line source):

$$\phi(x, y, z, 0) = 0 \qquad [4]$$

and

$$-\partial\phi/\partial z + \alpha\phi = 0, \qquad z = 0, \qquad x \neq 0. \qquad [5]$$

Also assume ϕ, $\partial\phi/\partial x$ and $\partial\phi/\partial z$ vanish as $x^2 + z^2 \to \infty$.

The solution of Eq. [3] with a three-dimensional point source of strength $q\Delta y$ at the origin buried in an infinite medium is given by Carslaw and Jaeger (1959, p. 261) is

$$\Delta\phi = q(\Delta y\sqrt{\alpha}/8\pi\sqrt{\pi k})\int_0^t \tau^{-3/2}\exp\{-\alpha[x^2 + y^2 + (z - k\tau)^2]/4k\tau\}\, d\tau. \qquad [6]$$

(Note: In the interest of conserving space, dummy variables of integration will not be defined in the text.)

To obtain a solution of Eq. [3] for a line source along the y-axis in an infinite medium, we consider q in the expression above to be the strength per unit time per unit length along the y-axis and integrate on y from $-\infty$ to ∞ to obtain

$$\phi = \int_{-\infty}^{\infty} q\sqrt{\alpha}/\,8\pi\,\sqrt{\pi k}$$

$$\int_0^t \tau^{-3/2}\exp\{-\alpha[x^2 + (z - k\tau)^2]/4k\tau\}\exp(-y^2/4k\tau)d\tau dy.$$

This integral is such that the interchange of the order of integration is permissible. Use of the well-known result (see, for example, Eq. 860.11 of Dwight, 1961)

$$\int_{-\infty}^{\infty}\exp(-a^2y^2)\,dy = \sqrt{\pi/a} \qquad [7]$$

gives the solution of Eq. [3] with a line source buried (subscripted by B) in an infinite medium:

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